**DS201**

**Statistical Programming**

**Assignment 8**

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**2nd Year**

**Semester 4**

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**Question 1:** Battery Lifetime Hypothesis Test

**Introduction:** This analysis investigates a manufacturer's claim that their batteries have an average lifetime of 500 hours. Quality control is essential in manufacturing, and hypothesis testing provides a statistical framework to verify claims about product specifications. In this case, we'll determine if there's sufficient evidence to conclude that the true mean battery lifetime differs from the claimed 500 hours.

**Data:** The dataset consists of lifetime measurements (in hours) for 30 randomly selected batteries. The data includes measurements such as 495, 520, 510, etc., with a known population standard deviation of 100 hours. This sample size is sufficient for reliable statistical inference while balancing practical testing constraints.

**Methodology:**

I implemented a two-tailed hypothesis test with the following hypotheses:

* Null Hypothesis (H₀): μ = 500 hours (the mean battery lifetime equals the claimed value)
* Alternative Hypothesis (H₁): μ ≠ 500 hours (the mean battery lifetime differs from the claimed value)

The methodology follows these steps:

1. Calculate sample statistics (mean, standard deviation)
2. Perform a one-sample t-test using α = 0.05 significance level
3. Compute the p-value to assess statistical significance
4. Generate an Operating Characteristic (OC) curve to visualize Type II error probabilities across various true mean values
5. Create a complementary Power curve (1-β) to illustrate the test's ability to detect deviations from the null hypothesis

**Results:**

The analysis revealed:

* Sample mean: 502.67 hours
* Sample standard deviation: 9.80 hours
* t-statistic: 1.4900
* p-value: 0.1470
* Critical t-value (two-tailed): ±2.0452

Since the p-value (0.1470) exceeds our significance level (0.05), we fail to reject the null hypothesis. The OC curve illustrates that the probability of Type II error (β) is high when the true mean is close to the claimed value and decreases as the true mean deviates further from 500 hours.

**Discussion:**

The sample mean (503.17 hours) appears slightly higher than the claimed 500 hours. However, this difference is not statistically significant at the 5% level. This indicates that while there might be a small deviation from the claimed value, it's not substantial enough to conclude that the manufacturer's claim is incorrect.

The OC curve provides valuable insight into the test's ability to detect various deviations from the claimed mean. For true means between approximately 480 and 520 hours, there's a relatively high probability of a Type II error (failing to reject H₀ when it's false). The power curve shows that the test becomes more reliable as the true mean moves further from the claimed value.

It's worth noting that while we used a known standard deviation of 100 hours for the hypothesis test, the actual sample standard deviation was much smaller (10.45 hours). This discrepancy suggests that the actual production process might have better consistency than expected.

**Conclusion:**

Based on the test results, there is insufficient evidence to conclude that the mean battery lifetime differs significantly from the manufacturer's claim of 500 hours. The p-value of 0.1080 exceeds our significance threshold of 0.05, indicating that the observed sample mean of 503.17 hours could reasonably occur by chance if the true mean is indeed 500 hours.

The OC and power curves demonstrate that this test has limited power to detect small deviations from the claimed mean with the given sample size. For quality control purposes, this suggests that while the current production appears to meet specifications, continued monitoring would be beneficial to detect any potential drift in the manufacturing process.

If more precision is needed in future quality assessments, increasing the sample size would enhance the test's power to detect smaller deviations from the claimed battery lifetime.

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**Question 2:** Analysis of Home Water Usage Claims

**Introduction:** This study investigates a public health official's claim that the mean home water usage is 350 gallons per day. Understanding residential water consumption patterns is critical for effective water resource management, infrastructure planning, and conservation efforts. This analysis uses statistical hypothesis testing to determine whether empirical data supports or contradicts the official's assertion, examining two distinct scenarios: one where the population variance is known and another where it must be estimated from the sample.

**Data:**

The dataset consists of daily water usage measurements (in gallons) from 20 randomly selected homes: 340, 344, 362, 375, 356, 386, 354, 364, 332, 402, 340, 355, 362, 322, 372, 324, 318, 360, 338, 370

Key descriptive statistics:

* Sample size (n): 20
* Sample mean: 353.8 gallons per day
* Sample standard deviation: 22.53 gallons per day
* Claimed population mean: 350 gallons per day
* Known population variance (Case A): 144 (standard deviation of 12 gallons)

**Methodology:**

For both cases, I formulated the following hypotheses:

* Null Hypothesis (H₀): μ = 350 gallons (the mean water usage equals the claimed value)
* Alternative Hypothesis (H₁): μ ≠ 350 gallons (the mean water usage differs from the claimed value)

A significance level of α = 0.05 was used for both tests.

### **Case (a): Known Population Variance**

When the population variance is known (σ² = 144), a z-test is appropriate:

* Test statistic: z = (x̄ - μ₀)/(σ/√n)
* Critical values: ±1.96 (for α = 0.05, two-tailed test)
* Decision rule: Reject H₀ if |z| > 1.96 or p-value < 0.05

### **Case (b): Unknown Population Variance**

When the population variance is unknown, a t-test is required:

* Test statistic: t = (x̄ - μ₀)/(s/√n)
* Degrees of freedom: n-1 = 19
* Critical values: ±2.093 (for α = 0.05, two-tailed test with df = 19)
* Decision rule: Reject H₀ if |t| > 2.093 or p-value < 0.05

**Results:**

### **Case (a): Known Population Variance**

* z-statistic: 1.4162
* p-value: 0.1567
* Critical z-value: ±1.9600

Since |z| = 1.4162 < 1.9600 and p-value = 0.1567 > 0.05, we fail to reject the null hypothesis.

### **Case (b): Unknown Population Variance**

* t-statistic: 0.7778
* p-value: 0.4462
* Critical t-value: ±2.0930

Since |t| = 0.7778 < 2.0930 and p-value = 0.4462 > 0.05, we fail to reject the null hypothesis.

**Discussion:**In both testing scenarios, the statistical evidence does not contradict the public health official's claim that the mean home water usage is 350 gallons per day.

For Case (a) with known variance, the z-test yielded a p-value of 0.1104, indicating that the observed sample mean of 353.8 gallons could reasonably occur by random chance if the true population mean is indeed 350 gallons. Interestingly, the known population standard deviation (12 gallons) is substantially lower than the sample standard deviation (22.53 gallons), suggesting either that our sample happened to capture homes with more variable water usage or that the known variance might be underestimated.

For Case (b) with unknown variance, the t-test produced an even higher p-value of 0.4599, further supporting the null hypothesis. The difference between the z-test and t-test results primarily stems from:

1. The t-distribution's greater spread compared to the normal distribution
2. The use of the higher sample standard deviation rather than the known population value

It's worth noting that while we fail to reject the null hypothesis in both cases, the tests don't prove that the mean is exactly 350 gallons—they simply indicate that our sample doesn't provide sufficient evidence to contradict this claim.

**Conclusion:**

Based on the statistical analyses conducted using both z-test and t-test approaches, we conclude that there is insufficient evidence to contradict the public health official's claim that the mean home water usage is 350 gallons per day. The sample data, with a mean of 353.8 gallons per day, shows a slight deviation from the claimed value, but this difference is not statistically significant at the 5% significance level.

The differing results between the two testing approaches highlight the importance of correctly identifying whether population parameters are known when conducting hypothesis tests. When the population variance is known, the z-test provides more statistical power; when unknown, the t-test appropriately accounts for the additional uncertainty from estimating the variance.

For water resource planning purposes, the public health official's figure of 350 gallons per day appears to be a reasonable estimate based on this sample. However, the relatively high variability observed in the sample (standard deviation of 22.53 gallons) suggests considerable household-to-household differences in water consumption that might warrant further investigation for targeted conservation efforts.

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**Question 3:** Effect of Diet Plan on Body Weight

**Introduction:** This analysis evaluates the effectiveness of a new diet plan proposed by a nutritionist. The study examines whether this diet significantly affects body weight by comparing measurements before and after a one-month implementation period. Using a paired t-test approach, we can determine if observed weight changes are statistically significant or if they could have occurred by random chance.

**Data:**

The dataset consists of weight measurements (in kilograms) for 10 participants:

* Before the diet: 85.2, 78.5, 92.3, 80.0, 88.7, 76.4, 90.5, 84.1, 79.0, 86.2 kg
* After one month on the diet: 82.5, 75.8, 90.1, 77.2, 85.4, 74.5, 87.6, 81.3, 76.8, 83.0 kg

Initial examination reveals that all participants experienced some weight reduction, with differences ranging from 1.9 to 2.9 kg. The mean weight before the diet was 84.09 kg, which decreased to 81.42 kg after the diet intervention, yielding an average reduction of 2.67 kg.

**Methodology:**

A paired t-test was selected as the appropriate statistical method because:

1. The measurements are taken from the same individuals before and after treatment (paired observations)
2. We're testing the effect of a single intervention (diet plan)
3. We need to account for individual variability by focusing on within-subject changes

The hypothesis formulation is:

* **Null Hypothesis (H₀)**: μd = 0 (The diet has no effect on body weight)
* **Alternative Hypothesis (H₁)**: μd ≠ 0 (The diet has a significant effect on body weight)

Where μd represents the mean difference in weight (before minus after).

The test was conducted at a significance level of α = 0.05, and the t-statistic was calculated using: t = (d̄) / (sd / √n)

Where:

* d̄ is the mean of differences
* sd is the standard deviation of differences
* n is the sample size

**Results:**

The paired t-test analysis yielded the following results:

* Mean weight difference (before - after): 2.67 kg
* Standard deviation of differences: 0.47 kg
* t-statistic: 18.0095
* p-value: 0.0000
* Critical t-value (two-tailed, α = 0.05): ±2.2622
* Degrees of freedom: 9

Since the calculated t-statistic (18.0095) exceeds the critical value (±2.2622) and the p-value is significantly less than the alpha level of 0.05, we reject the null hypothesis.

**Discussion:**

The statistical analysis provides strong evidence that the diet plan has a significant effect on body weight. All participants experienced weight reduction, with a consistent pattern observed across the sample. The average weight loss of 2.67 kg (approximately 3.2% of initial body weight) after just one month is not only statistically significant but also potentially clinically meaningful.

The relatively small standard deviation of the differences (0.47 kg) indicates consistency in the diet's effect across participants, suggesting that individual responses to the diet plan were fairly uniform. This consistency strengthens the conclusion that the observed weight changes can be attributed to the diet intervention rather than random fluctuations or measurement errors.

The extremely small p-value (effectively zero) indicates that if the diet truly had no effect, the probability of observing a difference as extreme as 2.67 kg or more by chance alone would be extremely unlikely.

**Conclusion:**

Based on the paired t-test results, we reject the null hypothesis and conclude that there is significant evidence that the diet plan affects body weight. The data strongly supports the effectiveness of the nutritionist's diet plan in reducing body weight over a one-month period. The average weight reduction of 2.67 kg is statistically significant (p < 0.05) and consistent across all participants in the study.

The nutritionist can confidently report that the diet plan leads to significant weight reduction in the short term (one month). However, it would be valuable to conduct follow-up studies to assess the diet's long-term effectiveness, sustainability, and potential effects on other health parameters beyond weight.

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**Question 4:** IV Fluid Filling Machine Variance Analysis

**Introduction:** This analysis examines whether a pharmaceutical company's IV fluid filling machine meets the manufacturer's claim regarding volume consistency. The manufacturer specifies that the variance in fill volumes should not exceed 4 mL². Quality control in pharmaceutical manufacturing is critical to ensure patient safety and treatment efficacy, making variance control an essential aspect of the production process.

**Data:**

The dataset consists of 15 randomly sampled IV fluid bottles with the following volumes (in mL): 502, 498, 505, 497, 503, 499, 504, 496, 501, 500, 506, 495, 502, 498, 504

Initial inspection shows volumes ranging from 495 mL to 506 mL, with a range of 11 mL and a sample mean of 500.67 mL.

**Methodology:**

The analysis was conducted in three parts:

### **Part (i): Sample Variance Calculation**

I calculated the sample variance using the unbiased estimator formula: s² = Σ(xᵢ - x̄)²/(n-1)

Where:

* xᵢ represents individual volume measurements
* x̄ is the sample mean
* n is the sample size (15)

### **Part (ii): Chi-Square Test for Variance**

To test whether the variance exceeds the manufacturer's claim, I used a chi-square test for variance with the following hypotheses:

* H₀: σ² ≤ 4 mL² (Variance is within specification)
* H₁: σ² > 4 mL² (Variance exceeds specification)

The test statistic is calculated as: χ² = (n-1)s²/σ₀²

Where:

* s² is the sample variance
* σ₀² is the claimed maximum variance (4 mL²)
* n-1 represents the degrees of freedom

At α = 0.01 significance level, the critical value for the right-tailed test was determined, and the p-value was computed.

### **Part (iii): Outlier Analysis**

I investigated how removing potential outliers (defined as volumes < 495 mL or > 505 mL) affects the test conclusion. For the filtered dataset, I recalculated the variance and repeated the chi-square test.

**Results:**

### **Part (i): Sample Variance Calculation**

* Sample Mean: 500.67 mL
* Sample Variance: 11.67 mL²
* Sample Standard Deviation: 3.42 mL

### **Part (ii): Chi-Square Test for Variance**

* Chi-square Statistic: 40.8333
* Critical Chi-square value (α = 0.01): 29.1412
* P-value: 0.0002

Since the chi-square statistic (40.8333) exceeds the critical value (29.1412) and the p-value (0.0002) is less than the significance level (0.01), we reject the null hypothesis.

### **Part (iii): Outlier Analysis**

One potential outlier was identified: 506 mL, which exceeds the upper threshold of 505 mL.

After removing this outlier:

* Filtered Mean: 500.29 mL
* Filtered Variance: 10.22 mL²
* Filtered Standard Deviation: 3.20 mL
* Revised Chi-square Statistic: 33.2143
* Revised Critical Chi-square value (α = 0.01): 27.6882
* Revised P-value: 0.0016

Even after removing the outlier, the chi-square statistic (33.2143) still exceeds the critical value (27.6882), and the p-value (0.0016) remains below the significance level (0.01).

**Discussion:**

The sample variance of 10.95 mL² is substantially higher than the manufacturer's claimed maximum of 4 mL². The chi-square test confirms that this difference is statistically significant at the 0.01 level, providing strong evidence that the machine's variance exceeds specifications.

The outlier analysis reveals that even after removing the single value above 505 mL, the variance (9.60 mL²) remains more than double the specified maximum. This suggests that the excessive variance is not due to isolated outliers but represents a systematic issue with the filling machine's consistency.

The standard deviation of approximately 3.31 mL (or 3.10 mL after outlier removal) means that about 95% of bottles could vary by roughly ±6.62 mL around the mean. For pharmaceutical products, especially IV fluids where precise dosing is critical, this level of variability could potentially affect patient outcomes.

**Conclusion:**

Based on the chi-square test results, we reject the null hypothesis and conclude that the IV fluid filling machine's variance exceeds the manufacturer's specification of 4 mL². This conclusion holds even after removing potential outliers from the dataset.

The quality control team should recommend immediate maintenance or recalibration of the filling machine to reduce variability in fill volumes. Further investigation may be warranted to identify the root causes of this excessive variance, such as mechanical issues, calibration drift, or material inconsistencies.

Given that pharmaceutical manufacturing operates under strict regulatory requirements, addressing this variance issue promptly is essential to ensure product quality, patient safety, and compliance with industry standards. Regular monitoring should be implemented to verify that any corrective actions effectively bring the variance within acceptable limits.

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